Cyclical characteristics of tithe series in Mid Frankonia and Switzerland 1339-1708: an application of maximum entropy spectral analysis

Bauernfeind, Walter; Woitek, Ulrich

Historical Social Research, Vol. 21 — 1996 — No. 1, 122-150

Cyclical Characteristics of Tithe Series in Mid Franconia and Switzerland 1339-1708: An Application of Maximum Entropy Spectral Analysis

Walter Bauernfeind and Ulrich Woitek*

Abstract: In this paper we present a method of describing the cyclical characteristics of a time series in the frequency domain by computing the spectrum of the series. Classical spectral estimation has serious deficiencies if used with economic data. For such short series, Maximum Entropy spectral estimation is more suitable. To demonstrate the method, we analyze the cyclical structure of tithe series from Germany and Switzerland in the period 1339-1708. These series are indicators for grain output. Grains were the most important agricultural products in the pre-industrial economy, providing no less than 70 per cent of human caloric requirements. Hence, fluctuations in output had implications for life and death. We find a robust cyclical structure in the tithe series with a cycle length of about 4 years. These cycles can be found in both the Mid Franconian data and the Swiss data. The coherence between cycles in different regions declines with geographical distance.

*** Address all communications to Walter Bauernfeind and Ulrich Woitek, Seminar for Economic History, Department of Economics, University of Munich, Ludwigstr. 33/rV, D-80539 München.** **We are indebted to the participants of the ZHSF workshop in Cologne (October 1995) for their helpful comments and suggestions, and to Brian A'Heam for his comments and for carefully reading the manuscript.**

1. Introduction

The term *spectrum* was introduced by Newton (1671), who used it to describe the band of light colors resulting from light passing through a glass prism, each color representing a particular wavelength of light.[1] The mathematical foundations of spectral analysis go back to D. Bernoulli, L. Euler, and J. B. J. Fourier. In the midth of the 19th century, spectral analysis techniques were applied to examine the periodicity of phenomena like sunspots, tides etc. The phenomena analyzed in these first applications exhibit obvious cyclical structure, which made it possible to describe them using the techniques available at that time. The analysis of series with hidden periodicities became feasible when Sir Arthur Schuster (1889) introduced the *periodogram* as an estimate for the spectrum of a time series. Beginning with Moore (1914), economists adopted this technique to examine the cyclical structure of economic time series, which was a widely accepted empirical finding at that time.

An overview of applications of spectral analysis in this early period can be found in Cargill (1974). He lists the work of Henry L. Moore (1914), (1923), William H. Beveridge (1921), William L. Crum (1923), Edwin B. Wilson (1934), Benjamin Greenstein (1935), and Harold T. Davis (1941). These early contributions, which analyse data sets of about 100-400 observations, are rather impressive given that today's technical possibilities for the computation of the spectral estimates were not available. After a revival in the 1950s and 1960s, interest in developing appropriate spectral analysis techniques and their application ceased. One of the reasons for this development is that classical spectral analysis techniques are not suited for economic time series, which are in general relatively short.[2]

In this paper we introduce a spectral estimation method which has already been fruitfully applied to the description of modern business cycle facts. This method is Maximum-Entropy (ME) spectral analysis, which was developed during the 1960s and 1970s in the natural sciences. It overcomes to a large extent the problems connected with the application of the periodogram to economic time series. ME-spectral estimation is especially suitable for econometric purposes, since it efficiently extracts the information on the cyclical structure contained in short time series, a problem which also occurs in economic history.[3]

---

[1] **This paragraph follows the description in Marple (1987), pp. 3-12.**

[2] **More recent examples for spectral analysis of historical economic time series can be found e.g. in Bengtsson and Joerberg (1975), Bengtsson and Ohlsson (1984), Metz (1988), and Eisner (1992).**

[3] **Examples for the application of ME-spectral analysis to empirical business cycle research can be found in Hillinger and Sebold-Bender (1992) and Woitek (1995).**

Knowledge of fluctuations of economic variables for the pre-industrial era is still limited, although contemporaries were aware of problems connected with the business cycle. We chose data on grain output for the years 1339-1708 as an example because of the importance of the agricultural sector in this period. In the pre-industrial world, this sector produced at least 75 per cent of »GNP« (Roesener 1984, p. 26). Grains were the most important agricultutral products: they provided no less than 70 per cent of human caloric requirements (Neveux 1979, p. 75). Hence, fluctuations in output had implications for life and death. As an indicator for grain output, tithe series are available. We analyze and compare the cyclical structure of the tithe in Mid Franconia and Switzerland. After presenting the data, we discuss problems of spectral analysis in econometrics. Then, we present the results to demonstrate the usefulness of the applied method and possible interpretations. The main focus of our study is the introduction of ME-spectral analysis.

2. Methodology and Data under Analysis

2.1 Estimation of the Spectrum

A useful method for describing the cyclical characteristics of a time series which can be represented by a linear model is to transform it from the time domain to the frequency domain and to compute the spectral density function which provides the measures needed for our analysis. We first give an overview of the spectral measures used in the paper. For a detailed description see Brockwell and Davis (1991), pp. 434-443, Priestley (1981), vol. II, and Koopmans (1974), pp. 119-164.

Univariate and Multivariate Spectral Measures

To obtain a first insight regarding the cyclical structure of a time series, usually the covariance functions of the series are computed.This function measures the degree of the linear relationship between two observations in the same series (autocovariances), or between two observations in different series (cross-covariances) at a lag x. High values of the autocovariance function at lag x mean that the series exhibits cyclical structure with a cycle length of about x units of time. Using the cross-covariance function we can analyze the lead-lag structure of two series. Although this information is very useful, it remains limited. For example, it is difficult to identify more than one cycle in the autocovariance function, and even if it were possible, one could hardly decide the relative importance of the cycles found in the series.[4]

*** See the example in Woitek (1995), p. 8ff.**

A much more powerful tool to describe the cyclical structure can be constructed by transforming the covariance function from the time domain into the frequency domain. This transformation is called *Fourier transformation,* and the resulting transform is the spectrum. The spectral density matrix $\mathbf{F}(\omega)$ of a n-dimensional stochastic process is the Fourier transform of the covariance function of the process, $\Gamma(\tau)$:

$$F(\omega)=\frac{1}{2\pi}\sum_{\tau=-\infty}^{+\infty}\Gamma(\tau)e^{-i\omega\tau};\qquad \omega=2\pi\lambda,\lambda\in[-0.5,0.5]; \tag{1}$$

with

$$\Gamma(\tau)=\begin{pmatrix}\gamma_{11}(\tau) & \cdots & \gamma_{1n}(\tau)\\ \vdots & \ddots & \vdots\\ \gamma_{n1}(\tau) & \cdots & \gamma_{nn}(\tau)\end{pmatrix};\ F(\omega)=\begin{pmatrix}f_{11}(\omega) & \cdots & f_{1n}(\omega)\\ \vdots & \ddots & \vdots\\ f_{n1}(\omega) & \cdots & f_{nn}(\omega)\end{pmatrix};$$

Since $\mathbf{F}(\omega)$ is an even function, it is sufficient to look at it in the interval $[0,\pi]$. The diagonal elements $f_{11}(\omega)$, . . ., $f_{nn}(\omega)$ are the real valued *autospectra* or *power spectra* of the *n* individual series:

$$f_{jj}(\omega)=\frac{1}{2\pi}\left(\gamma_{jj}(0)+2\sum_{\tau=-\infty}^{+\infty}\gamma_{jj}(\tau)\cos(\omega\tau)\right);\ j=1,\dots,n; \tag{2}$$

The area under the power spectrum equals the process variance $\gamma_{jj}(0)$:

$$\gamma_{jj}(0)=\int_{-\pi}^{\pi}f_{jj}(\omega)d\omega; \tag{3}$$

In this paper, we use the normalized power spectrum, i.e. the power spectrum $f_{jj}(\omega)$ is divided by the process variance $\gamma_{jj}(0)$:

$$\tilde{f}_{jj}(\omega)=\frac{f_{jj}(\omega)}{\gamma_{jj}(0)}; \tag{4}$$

Hence, the area under the normalized power spectrum equals one. If one assumes that the autospectrum $f_{jj}(\omega)$ has a peak at frequency ω^*, then the expression

$$pp(\omega^*) = \frac{2}{\gamma_{jj}(0)} \int_{\omega^*-0.1\omega^*}^{\omega^*+0.1\omega^*} f_{jj}(\omega)d\omega \tag{5}$$

can be interpreted as the part of the variance $\gamma_{jj}(0)$, which is explained by the variance of oscillations with frequencies in the range ± 10 per cent around the peak frequency ω^*. In the following, this expression is called *peak power (pp)* of a cycle with the frequency ω^*.

The spread of a peak, i.e the damping of a cycle, is measured by the *bandwidth (bw),* defined as the range in which the peak halves: The sharper the peak at a frequency ω^*, the smaller the bandwidth (See Priestley 1981, pp. 513-517). The bandwidth has the disadvantage that it cannot be computed if the respective cycle is too strongly damped or if two peaks are too close to one another. Therefore, more information is obtained by looking at the moduli of the corresponding complex roots of the characteristic polynomial of the AR-model used to estimate the univariate spectrum as explained below.

Another important measure is the *signal-to-noise ratio.* It measures the influence of the noise on a series j and is defined as the ratio of the variance of the signal to the variance of the noise.

$$SNR = \frac{\int_{-\omega}^{\omega} f_{jj}(\omega)d\omega - \sigma^2_{uj}}{\sigma^2_{uj}}; \tag{6}$$

The elements
real valued, s
Therefore, f_{jk}

$$f_{jk} = \frac{1}{2\pi} \sum_{\tau=-\infty}^{+\infty} \gamma_{jk}(\tau) e^{-i\omega\tau} = c_{jk}(\omega) - iq_{jk}(\omega); \quad j=1,\ldots,n;\ k=1,\ldots,n;\ j \neq k; \tag{7}$$

$$f_{jk} = \frac{1}{2\pi} \sum_{\tau=-\infty}^{+\infty} \gamma_{jk}(\tau) e^{-i\omega\tau} = c_{jk}(\omega) - iq_{jk}(\omega); \quad j=1,\ldots,n;\ k=1,\ldots,n;\ j \neq k; \tag{7}$$

where $c_{jk}(\omega)$ is the *cospectrum* and $q_{jk}(\omega)$ is the *quadrature spectrum.* From the co- and the quadrature spectrum of two series j and k it is possible to compute measures for the lead-lag relationships between them. These measures are the *phase spectrum* and the *squared coherency.*

The squared coherency can be interpreted in the same way as the correlation coefficient in a regression model. It measures the degree of linear relationship between a cycle of frequency ω in the series j and a cycle of the same frequency in the series k. If it equals 1, there is an exact linear relationship between the cycles with frequency ω in the two series; if it equals 0, there is no relationship between the two cycles. The squared coherency is defined as

$$|\kappa_{jk}(\omega)|^2 = \frac{|f_{jk}(\omega)|^2}{f_{jj}(\omega) f_{kk}(\omega)}; \tag{8}$$

The phase spectrum can be interpreted in the same way as the impact of a univariate linear filter on an input series in the frequency domain:

$$\phi_{jk}(\omega) = -\arctan(q_{jk}(\omega)/c_{jk}(\omega)) \tag{9}$$

measures the phase lead of the series j over the series k at a frequency ω. If the squared coherency $|\kappa_{jk}(\omega)|^2$ equals 1, there is a fixed linear relationship between the two series at the frequency ω. If it is less than 1, the phase and the gain have to be interpreted as expected values. Therefore, it is only sensible to look at lead-lag relationships of cycles where the squared coherency reaches values near 1.

Spectral Estimation

To estimate the spectral density matrix, the theoretical covariance function $\Gamma(\tau)$ is inferred from the estimated covariance function, $\Gamma(\tau)$ is which is only known for $|\tau| \leq N-1$. Classical spectral analysis solves this problem by assuming that the covariances are zero outside the observation period N. This method has well known defects (see e.g. the discussion in Koopmans 1974, pp. 294–336): The unsmoothed empirical spectrum, i.e. the periodogram is an asymptotically unbiased but inconsistent estimate for the true spectral density matrix. A consistent estimate can be found by smoothing it using spectral windows, but this method has the disadvantage that it reduces resolution, i.e. the ability to distinguish between peaks in the spectra. These drawbacks lead to problems in the interpretation of the information from the (smoothed or unsmoothed) periodogram.

A preferable method to estimate the spectrum of a time series was developed by Burg (1975), using the Maximum-Entropy Principle (or First Principle of Data Reduction) by Shannon and Jaynes: *The result of any transformation imposed on the experimental data shall incorporate and be consistent with all relevant data and be maximally non-committal with regard to unavailable data.* (see Ables 1974, p 23)

Applying this principle to spectral estimation, one has to choose the spectrum which is non-informative concerning out-of-sample data structure, subject to the restriction that the resulting spectrum has to be compatible with the Fourier transform of the first p sample correlations, i.e. has to correspond to the inner-sample information. The resulting Maximum-Entropy (ME-) spectrum is equivalent to the spectrum of an AR(p)-process, or, in the multivariate case, a VAR(p)-process, for which the parameters are defined by an equation system which is formally identical to the Yule-Walker equations (Priestley 1981, pp. 604-606). In equations (10) and (11) definitions of an AR- and VAR-process and their spectra are given.

Univariate case

$$\begin{aligned} X_t &= \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \ldots + \alpha_p X_{t-p} + u_t; \\ u_t &\sim NID(0, \sigma^2_u) \\ \alpha(\omega) &= 1 - \alpha_1 e^{-i\omega} - \alpha_2 e^{-i2\omega} - \ldots - \alpha_p e^{-ip\omega}; \\ f(\omega) &= \frac{1}{2\pi} \frac{\sigma^2_u}{|\alpha(\omega)|^2}; \end{aligned} \tag{10}$$

Multivariate case

$$\begin{aligned} \boldsymbol{X}_t &= \boldsymbol{A}_1 \boldsymbol{X}_{t-1} + \boldsymbol{A}_2 \boldsymbol{X}_{t-2} + \ldots + \boldsymbol{A}_p \boldsymbol{X}_{t-p} + \boldsymbol{u}_t; \\ \boldsymbol{u}_t &\sim NID(\boldsymbol{0}, \boldsymbol{\Sigma}_u); \\ \boldsymbol{A}(\omega) &= \boldsymbol{I} - \boldsymbol{A}_1 e^{-i\omega} - \boldsymbol{A}_2 e^{-i2\omega} - \ldots - \boldsymbol{A}_p e^{-ip\omega}; \\ \boldsymbol{F}(\omega) &= \frac{1}{2\pi} \boldsymbol{A}(\omega)^{-1} \boldsymbol{\Sigma}_u \boldsymbol{A}(\omega)^{-*}; \end{aligned} \tag{11}$$

where X, is a $(n \times 1)$ vector, $\mathbf{A}_j$., $j = 1, \ldots, p$ are the $(n \times n)$ parameter matrices of the VAR(p)-process, and the superscript "*" denotes the complex conjugate transpose.

The estimation algorithms used in our paper are based on the fact that time direction is not important in the frequency domain, i.e. the parameters or parameter matrices are chosen so as to minimize both the forward- and the backward prediction errors. These algorithms have proved to be superior to the ordinary least squares algorithm (see Swingler 1979).[5]

A problem which can arise in multivariate ME-spectral estimation is the so called feed-across effect This effect describes the fact that peaks that should be present only in one of the series' autospectra can be found in the autospectra of the other series, too. As Marple and Nuttal (1983) show, the feed-across effect is unavoidable. They propose to use the multivariate spectra only to estimate the lead-lag relationships between the series. For describing the cyclical characteristics of single series it is better to look at the univariate spectral estimate than to analyze the results of the autospectra derived from a multivariate estimate.

[5] In the univariate case, we use the *Burg-algorithm* (Burg 1975), and alternatively, the *Fougere-algorithm* (Fougere 1985). In the multivariate case, the *Vieira-Morf algorithm* is applied (Vieira and Morf (1978)). For a more detailed explanation of the univariate algorithms and their application in macroeconometrics see Hillinger and Sebold-Bender (1992). A description of the Vieira-Morf algorithm and a comparison with other estimators can be found in Marple (1987), pp. 404-405, 409-416.

We assumed above that the order p of the filter is known; but in practice, we have to estimate it. The information criteria usually applied to solve this problem have well known drawbacks. Therefore, we decided to use the BAC-criterion recently developed by Heintel (1994). This criterion is based on the framework of Bayesian time series analysis. Applying this concept it is possible to use prior information about the order of the process, which can be very easily modelled, since it results in a discrete distribution.

The BAC is preferable to the widely used information criteria from a theoretical point of view, because these methods determine the order by minimizing the sum of the estimated error variance and a more or less arbitrary penalty term, and because prior information can be taken into account in a way which is replicable. Preliminary results of a simulation study by Markus Heintel show that the BAC is also preferable from a more practical view, because it results in more robust order estimates. The criterion is based on the continous version of Bayes' theorem (Judge, Hill, Griffiths, Lütkepohl and Tsuong-Chao 1988, pp. 117-120):

$$f(\boldsymbol{\theta}|\mathbf{y}) \propto L(\boldsymbol{\theta}|\mathbf{y}) f(\boldsymbol{\theta});$$

where $\boldsymbol{\theta}$ is the parameter vector to be estimated, y is the vector of observations, $f(\boldsymbol{\theta})$ is our prior distribution of the parameter vector $\boldsymbol{\theta}$, $L(\boldsymbol{\theta}|\mathbf{y})$ is the likelihood function, and $f(\boldsymbol{\theta}|\mathbf{y})$ is the resulting posterior distribution of $\boldsymbol{\theta}$.

The non-informative case is the relevant one for our study. In this case, one has to assume as prior information that the order is equally distributed in the range 0, ... h where h is the highest possible order to be determined before. The resulting posterior distribution is

$$f(p|\mathbf{y}) \propto \frac{f(p)\Gamma^{*}\dfrac{a^{(p)}_{1}}{2}}{(2\pi)^{a^{(p)}_{1}/2}|\boldsymbol{F}^{(p)}|^{1/2}\left(\dfrac{b^{(p)}_{1}}{2}\right)^{a^{(p)}_{1}/2}};\ p=0,\ldots h; \qquad \textbf{(12)}$$

with

$$\begin{aligned}
h &: \textit{highest possible order}\\
f(p) &= 1/(h+1);\\
a^{(p)}_{1} &= N-p-1;\\
\boldsymbol{F}^{(p)} &= \boldsymbol{X}^{(p)T}\boldsymbol{X}^{(p)};\\
b^{(p)}_{1} &= \mathbf{y}^{T}\left(\boldsymbol{I}-\boldsymbol{X}^{(p)}(\boldsymbol{F}^{(p)})^{-1}\boldsymbol{X}^{(p)T}\right)\mathbf{y};
\end{aligned}$$

$\Gamma^*(.)$ denotes the gamma function, $\mathbf{y} = (y_1, y_2, ..., y_N)^T$ is the vector of observations and $\mathbf{X}^{(p)}$ is the $(N \times (p+1))$-matrix consisting of N rows with the value 1 and the p lagged observations per time t, t - 1, ... N.[6] The BAC-estimate of the order p is the mode of the posterior distribution $f(p \mid \mathrm{y})$.

Since a multivariate extension of the BAC is not yet available, we have to make a reasonable choice from the large number of widely used information criteria to estimate the order of a VAR-process. In simulation studies to judge the performance of different information criteria, Liitkepohl (1985) and (1991), pp. 135-139, finds that for data samples generated by low order VAR-processes, very parsimonious methods such as the multivariate Schwarz criterion lead to better results than other criteria. However, in practice it could be the case that the unknown data generating process is of infinite order and has to be approximated by a finite order model. In this case, less parsimonious criteria such as the multivariate CAT (criterion for autoregressive transfer functions) might perform better. Therefore, Liitkepohl recommends a comparison of the results for VAR-processes of different order estimates. We estimate the order of the VAR-models using the multivariate versions of Akaike's information criterion (AIC), the Schwarz criterion (BIC), the Hannan-Quinn criterion, and the CAT (Liitkepohl 1985) and compare the resulting autospectra with the univariate results. We chose the most parsimonious order for which the twospectra show similar characteristics.

2.2 The Data

We now apply the above described methodology to tithe series as indicators for grain production. Based on the statistics, it is for example possible to explore the effect of supply shocks on prices (see e.g. Goy and Le Roy Ladurie 1972, 1982a, 1982b). Institutions such as churches, monasteries, hospitals, etc. had the privilege to collect a fixed percentage (not always 10 per cent) of the annual crop of specified areas under cultivation. As a rule, these institutions did not collect the tithe themselves, but auctioned the right to collect it just before the harvest, after having estimated the crop using their own experts. Bids were made in terms of the quantity of threshed grain to be delivered. These deliveries, which varied from year to year, were often recorded in tithe registers. On the basis of tithe yields, one can obtain a reliable series reflecting annual fluctuations in output as well as long-term production trends. The analyzed tithe privileges must remain the same percentages of the crop yield over the centuries and the series have to cover different tithe districts with high quantities (Goy and Le Roy Ladurie 1982b, pp. 3-67). Our series meet these requirements: they are complete and representative of the regions under analysis.

---

[6] The starting values $y_{-p+1}, \ldots, y_0$ are the first p observations.

We have chosen seven series for Germany and Switzerland. For Germany only one series is available representing the Nuremberg market area covering the production of between 43 and 156 villages in the observation period 1339-1670 (Figure 1). Most of these villages are in the area of the present Bavarian administrative district *Mittelfranken* (Mid Franconia). Hence, for the sake of simplicity, we will call the area Mid Franconia.[7]

The Swiss tithe series come from the *Schweizer Mittelland* between Lake Geneva and Zurich: *Waadt, Seeland, Berner Mittelland* (Region Bern), *Emmental, Unteraargau* (Aargau), and Zurich. Each of these regions is represented by two sub-regions Pfister (1985), Vol. II, pp. 65-80). Since the series do not have the same starting date, we chose for the Swiss data set the observation period 1558-1708 (See Figure 2).[8]

2.3 The Problem of Stationarity

For the application of spectral analysis we need (weakly) stationary time series, i.e. series for which the mean is constant and the covariance function is a function of the lag x but not of time. In the first part of the section, it was assumed that the series we are analyzing have this property. But a look at the plot of the tithe series (Figures 1 and 2) shows that these series are apparently not stationary, neither with respect to the mean nor to the variance. Moreover, a structural break in the Mid Franconian series is clearly to be seen. Hence, in order to separate the long-run and the medium-run fluctuations correctly, we have to find an appropriate detrending method.

It is a common practice to homogenize the variance by simply taking logarithms.[9] However, there is no such easy transformation available to solve the problem of a non-stationary mean. There are two types of non-stationarity: the trend stationary (TS-) model and the difference stationary (DS-) model. In equations (13) and (14), two simple examples of these models are given.

---

[7] Until 1385 only the tithe records of one monastery to the West of Nuremberg are extant which include the tithe yields of 43 villages. From 1386 on, statistics of three other large districts to the West and North of Nuremberg are available (65 villages), but in 1392 the series of the above mentioned monastery ends, resuming only in 1554. From 1427 on, a fourth series becomes available (18 villages), and after about 1545, seven tithe districts are extant with 113 villages. These tithe series have been combined to a cumulative curve by extrapolations of missing values (for further information see Bauernfeind (1993), pp. 84-97, the data can be found on pp. 482-497).

[8] For about the same period, tithe data from France are also available (Goy and Le Roy Ladurie (1972), (1982a), 1982b)). But these series are too incomplete to be useful in our context.

[9] All series analyzed in the following sections are in logarithms.

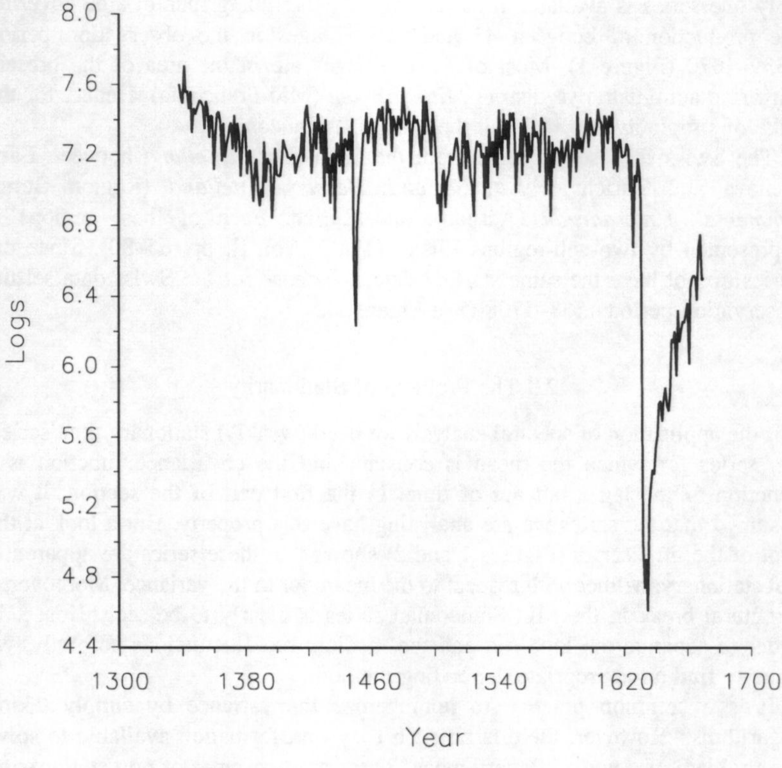

Figure 1: Tithe in Mid Franconia, 1339-1670

$$y_t = \alpha + \beta t + \zeta_t; \tag{13}$$

$$(1 - L) y_t = \zeta_t; \tag{14}$$

L denotes the backshift operator ($L^p y_t = y_{t-p}$), and $\{\zeta_t\}$ is assumed to be an arbitrary stationary stochastic process.

The linear time trend in equation (13) can be removed by the estimation of the parameters α and β and substracting the resulting trend. The non-stationarity of the DS-model in equation (14) is due to integration of order 1 (I(1)). To obtain the stationary process $\{\zeta_t\}$, one has to use a first order difference filter.

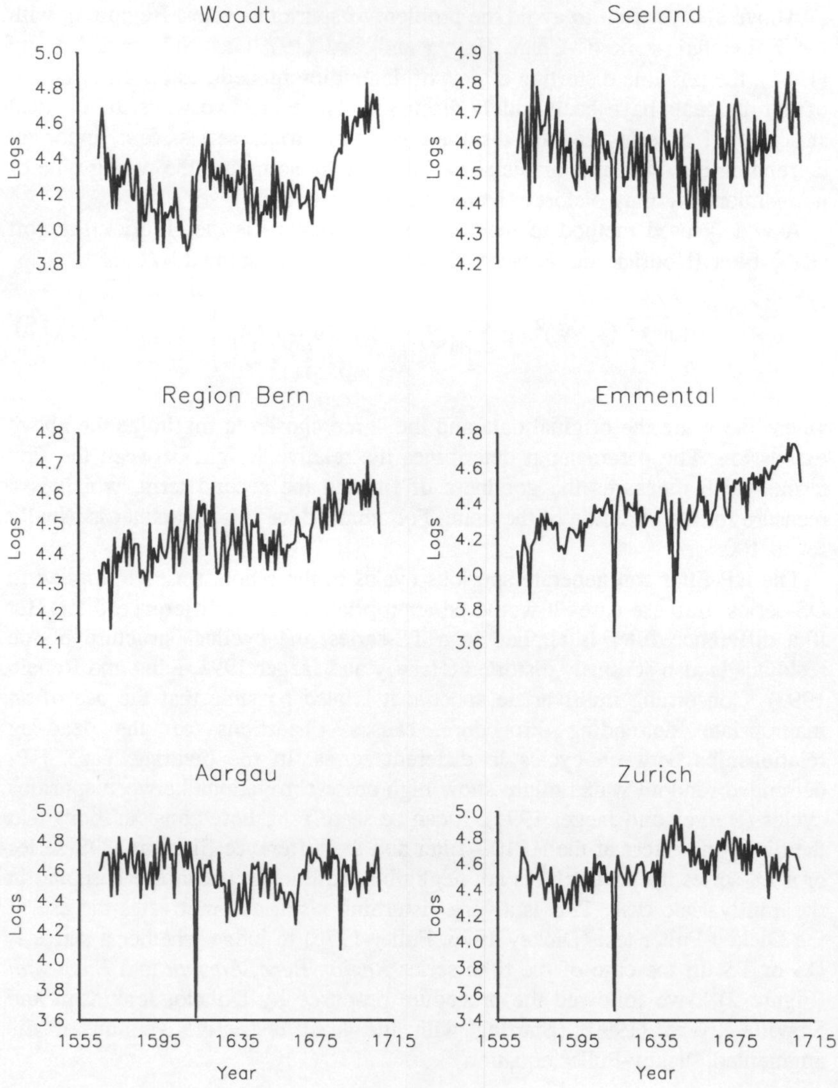

Figure 2: Tithe in Switzerland, 1558-1708

Above all, we want to avoid the problem of spurious cycles. Beginning with the influential work of Chan, Hayya and Ord (1977) as Nelson and Kang (1981), the possible distorting effects of detrending procedures on the structure of the residuals have been widely discussed. There are two ways the cyclical structure of a series can be distorted in a univariate series: first, incorrect detrending may generate artificial cycles. Second, assuming the wrong type of non-stationarity may distort existing cyclical structure.

A widely used method to make a series stationary is the Hodrick-Prescott (HP-) filter (Hodrick and Prescott (1980)),[10] which is defined by

$$\min_{\tilde{y}_t}\left(\sum_{t=1}^{N}(y_t-\tilde{y}_t)^2+\mu\sum_{t=2}^{N-1}((\tilde{y}_{t+1}-\tilde{y}_t)-(\tilde{y}_t-\tilde{y}_{t-1}))^2\right) \tag{15}$$

where the y_t are the original data and the $\tilde{y}_t$ are chosen to minimize the above expression. The parameter μ determines the relative weight between the first term, which measures the goodness of fit, and the second term, which is a measure for the variation of the trend. For annual data, this parameter is usually set to 100.

The HP-filter can generate spurious cycles in the output series if applied to DS-series. In these cases it would be appropriate to use a difference filter. But if a difference filter is applied to a TS-series, the cyclical structure of the residuals is also seriously distorted (Harvey and Jaeger 1991, King and Rebelo 1993). Concerning multivariate spectra, it is also possible that the use of an inapropriate detrending procedure causes distortions of the lead-lag relationships between cycles in different series. In the bivariate case, HP-detrended random walks might show high cross-correlations between spurious cycles (Harvey and Jaeger 1991). It can be seen from these considerations that the distorting effect of the HPlOO-filter and the difference filter on a DS-series or a TS-series, respectively, is not negligible, both in the univariate case and in the multivariate case. This is a very disturbing result and motivates the use of the Dickey-Fuller test (Dickey 1976, Fuller 1976) to judge whether a series is DS or TS. In the case of the tithe series *Region Bern, Aargau* and *Emmental* (Figure 2)[11], we followed the procedure proposed by Dolado, Jenkinson and Sosvilla-Rivero (1990): Starting with the least restrictive version of the augmented Dickey-Fuller equation

$$y_t=\mu+\beta t+\rho_1 y_{t-1}+\sum_{j=2}^{p}\rho_j(1-L)y_{t-j+1}+u_t; \tag{16}$$

[10] For recent applications in of the HP-filter in the description of business cycle stylized facts see e.g. Backus and Kehoe (1992) and Brandner and Neusser (1992).

[11] The extreme outliers in the year 1614 in the series *Aargau, Emmental,* and *Zürich* are treated as missing values.

we tested successively the null hypothesis of a unit root and the signifcance of the other parameters.[12] ρ_1 is the sum of the AR-parameters,[13] which, in the case of a unit root, i.e. in the case of a DS-model, have to be equal to 1. Therefore we have to test the hypothesis H_0: $\rho_1 = 1$ against the alternative H_1: $|\rho_1| < 1$ by comparing the statistic $\tau = (\rho_1 - 1) / \sigma_{\rho 1}$ with the critical values.[14] If the null hypothesis of a unit root cannot be rejected, one would prefer to use a difference filter in order to avoid generating spurious cycles.

However, the Dickey-Fuller test has only low power against plausible TS-alternatives.[15] Perron (1989) has shown that the DF-test becomes even more biased in favour of the unit root hypothesis if the data generating process contains real world phenomena such as structural breaks. The plot of the Mid Franconian tithe series (Figure 1) and the tithe series from *Waadt, Seeland,* and, after replacing the outlier in 1614 with a missing value, from *Zurich* (Figure 2) indicate that there are obvious structural breaks in the series, which cause changes in both the level and the slope of the trend. In these cases it is more appropriate if we apply the augmented unit root test proposed by Perron (1989), which allows for structural breaks in the alternative model. The model *(Mixed-Type model)* to be estimated is

$$y_t = \mu + \beta t + \theta DU_t + \gamma DT_t + \rho_1 y_{t-1} + \sum_{j=2}^{p} \rho_j (1-L) y_{t-j+1} + u_t; \tag{17}$$

$$t = 1,\dots,T_b,\dots,N;$$

with

$$DU_t = \begin{cases} 1 & \text{if } t > T_b \\ 0 & \text{if } t \le T_b \end{cases}; \quad DT_t = \begin{cases} t & \text{if } t > T_b \\ 0 & \text{if } t \le T_b \end{cases};$$

where T_b denotes the time of break.[16] Again, we have to test the hypothesis H_0: $\rho_1 = 1$ against the alternative H_1: $|\rho_1| < 1$. This can be done by comparing the τ-statistic for ρ_1 with the critical values given in Perron (1989), p. 1377, Table

[12] The maximum lag p of the AR-part is chosen so that the hypothesis that the residuals are white noise cannot be rejected using the Q^* -statistic (see Ljung and Box (1981)).

[13] $\rho_k = \Sigma_{p,j=k}\, \alpha_j$; k = 1, . . ., p.

[14] The critical values are given in Fuller (1976), p. 373, Table 8.5.2, and Dickey and Fuller (1981), Tables I, B, III, p. 1062.

[15] See Rudebusch (1992) and (1993) for an overview of the recent research on the power of the Dickey-Fuller test.

[16] The time of break is estimated using the CUSUM and CUSUMSQ-statistics (see e.g. Harvey (1990), p. 153-155.

VI. B.[17] The results of the unit root tests are given in Table 1. We see from Table 1 that the unit root hypothesis can be rejected for the entire data set. Hence, we decided to use the HP filter to make the series stationary. Moreover, this result indicates that we do not have to take into account the problem of cointregation for the analysis of the multivariate cyclical structure in Section 3.2 below.

Table 1: Tithe Series: Unit Root Tests

Series	Model	$(\hat{\rho}_1 - 1)/\hat{\sigma}_{\hat{\rho}_1}$	$H_0 : \rho_1 = 1$
Mid Franconia	Mixed-Type Model	-6.11	rejected ($\lambda = 0.9$)
Waadt	Mixed-Type Model	-7.39	rejected ($\lambda = 0.3$)
Seeland	Mixed-Type Model	-4.31	rejected ($\lambda = 0.8$)
Region Bern	ADF-Model (Drift+Trend)	-6.57	rejected ($N = 145$)
Emmental	ADF-Model (Drift+Trend)	-4.43	rejected ($N = 144$)
Aargau	ADF-Model (Drift)	-3.59	rejected ($N = 136$)
Zürich	Mixed-Type Model	-8.22	rejected ($\lambda = 0.3$)

Notes:

N: sample size; λ: time of break / sample size

H_0 is rejected in every case at least at the 5 per cent level.

3. The Tithe Spectrum

3.1 Univariate Results

We start our analysis with the Mid Franconian tithe series. Because of the severe structural break in 1632, we decided to restrict the observation period to 1339-1630. Otherwise, the outliers in the years after 1632 would have distorted the results. For the HP-filtered tithe series of Mid Franconia, BAC estimates that the AR-model is of order 6. Fitting the AR-model yields estimates of the model parameters and the error variance, which, applying equation (10), result in the ME-spectrum for the HP-filtered tithe series in Figure 3 (solid line). As mentioned above, there is a problem concerning the reliability of unit root tests. To test the robustness of our results, we therefore fitted an AR-model of the same order to the difference filtered series and computed the respective ME-spectrum. The resulting spectrum can be seen in Figure 3 (dashed line). The estimated cycle lengths, peak powers, bandwidths, and signal-to-noise ratios are displayed in Table 2.

[17] The critical values of the Perron-test are not plotted against the sample size, but against X, the ratio of the time of break to total sample size.

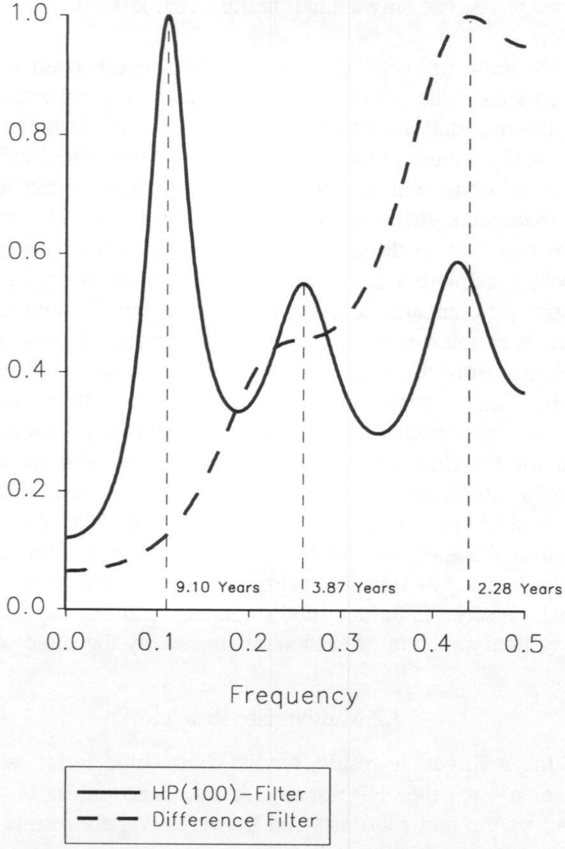

Figure 3: Mid Franconia: Univariate Tithe Spectra, HP-Filter and Difference Filter

We see that the cyclical structure changes remarkably dependent on the filtering method used. In the HP-filtered series, three cycles can be found with lengths of about 9 and 2-4 years. The shorter cycles have a higher peak power (0.12-0.20),[18] i.e. they are more important than the long cycle. In the difference filtered series, the spectrum has most of its power at the higher frequencies: Only the very short cycle can be seen with a peak power of 0.36. The longer cycle completely vanished, and the 4 year cycle exhibits no clear peak like for

[18] **I.e. the** 12-20 **per cent of the variance of the** HP-**filtered series can be explained by the variance of the cycles.**

the HP-filtered series, but only a small hump at the left side of the peak of the shortest cycle.

The signal-to-noise ratio is higher for the difference filtered series than for the HP-filtered series. These differences show clearly the importance of correct treatment of the non-stationarity of a time series. As result it can be said that there is a cyclical structure in the Mid Franconian tithe series, and that at least the short cycle is robust with respect to the chosen filtering method.

In order to ascertain whether this results for Mid Franconia were typical for the time, we now turn to the analysis of tithe series from Switzerland. The estimated spectra are plotted in Figure 4. The spectral measures can be found in Table 2. Again, we compare the outcome for both the HP- and the difference filtered series. It is noteworthy that the cyclical structure is similar to the Mid Franconian series. Both the long and the short cycles can be observed, but the short cycle has higher peak power and lower moduli than the long cycle. Consequently we can conclude that the results of the Mid Franconian tithe are quite typical for the time period. Again we see that with the exception of *Seeland, Region Bern,* and *Aargau,* the difference filtered series exhibits different cyclical characteristics than the HP-filtered data. The fact that the characteristics of the spectrum for the difference filtered series are strikingly similar (cycle length: 2-3 years; peak power of about 0.4) might be an indication that the series is in fact trend stationary, and that the difference filter distorts the cyclical structure of each series in exactly the same way.

3.2 Multivariate Results

To compute the multivariate results for the Swiss tithe series, we decided to restrict ourselves to the HP-filtered series, since this is the method recommended by the unit root tests and by the univariate results from above. The results are displayed in Table 3.

Table 3 is read as follows: We estimated a VAR-model containig all 6 series with order 4 as proposed by the CAT-criterion. As result, we obtain a (6×6) spectral density matrix $F(\omega)$ for each frequency ω, $\omega \in [-\pi, \pi]$ (see equation 11). Then we computed for each pair of regions from the elements of $F(\omega)$ the multivariate spectral measures as explained in Section 2.1 above.

Consider the results for *Waadt/Seeland.* In Figure 5 the two autospectra (Figures 5a and 5b), the coherence spectrum (Figure 5c), and the phase spectrum (Figure 5d) are plotted.[19] The first line (Max. of Coh.) contains the maximum squared coherency *(msc),* the phase and the gain computed at the maxima of the coherence spectrum (see Figure 5c). We see that the coherence spectrum exhibits two peaks at the cycle lengths 4.46 and 2.98 years with *mscs* of about 0.4. The phase shift (see Figure 5d) is less than a year, hence, with

[19] In order to save space, we do not plot the gain spectrum. The results for the gain can be found in Table 3.

Table 2: Tithe Series in Mid Franconia and Switzerland, Univariate Spectra

	Trend	Period	*pp*	*bw*	Moduli	SNR	AR-order
Mid Franconia	HP(100)	9.10	0.10	5.59	0.74	4.41	6
		3.87	0.12	-	0.83		
		2.35	0.20	-	0.83		
	Diff.	2.28	0.36	-	0.59	5.88	4
Waadt	HP(100)	-	-	-	-	4.19	1
	Diff.	2.80	0.37	1.19	0.67	7.55	2
Seeland	HP(100)	8.10	0.06	-	0.82	5.49	7
		4.12	0.12	-	0.84		
		2.81	0.31	0.61	0.84		
	Diff.	2.84	0.39	0.80	0.77	8.26	5
Region Bern	HP(100)	6.11	0.12	3.74	0.77	4.82	4
		2.65	0.26	0.88	0.77		
	Diff.	5.29	0.08	-	0.78	7.43	5
		2.64	0.38	0.79	0.78		
Emmental	HP(100)	9.24	0.11	5.00	0.79	4.83	6
		4.16	0.11	-	0.85		
		2.26	0.26	-	0.85		
	Diff.	-	-	-	-	5.62	1
Aargau	HP(100)	4.05	0.14	-	0.76	4.82	6
		2.51	0.26	-	0.76		
	Diff.	2.58	0.37	0.92	0.72	7.77	4
Zurich	HP(100)	-	-	-	-	4.21	1
	Diff.	2.33	0.40	-	0.51	7.03	2

Notes:
pp : Peak Power
bw : Bandwidth
SNR : Signal-to-Noise Ratio

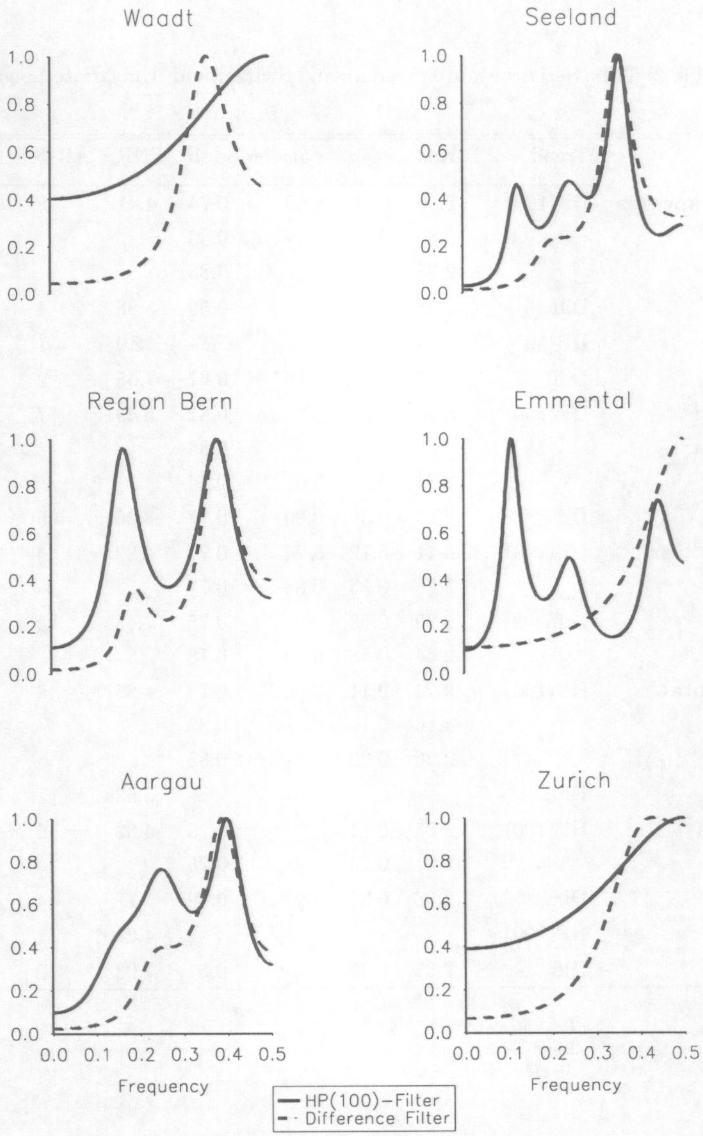

Figure 4: Switzerland: Univariate Tithe Spectra,

HP-Filter and Difference Filter

Table 3: HP-filtered Series, Multivariate Spectra

	Period	*msc*	Phase	Gain	VAR-Order
Waadt/Seeland					
Max. of Coh.	4.46	0.45	0.13	0.69	4
	2.98	0.43	-0.18	0.64	
Spectrum Waadt	11.24	0.02	-1.15	0.25	
	6.45	0.34	0.99	0.57	
	2.87	0.41	-0.13	0.59	
Spectrum Seeland	6.90	0.28	1.10	0.50	
	4.72	0.44	0.25	0.66	
	2.81	0.40	-0.10	0.56	
Waadt/Region Bern					
Max. of Coh.	6.49	0.25	0.74	0.51	4
	3.41	0.25	0.06	0.97	
Spectrum Waadt	11.24	0.06	0.19	0.56	
	6.45	0.25	0.74	0.51	
	2.87	0.23	-0.01	0.95	
Spectrum Bern	5.92	0.22	0.75	0.45	
	2.61	0.31	0.05	0.88	
Waadt/Emmental					
Max. of Coh.	6.49	0.10	1.41	0.28	
	3.47	0.31	0.62	1.05	
Spectrum Waadt	11.24	0.13	-1.12	0.39	
	6.45	0.10	1.39	0.28	
	2.87	0.18	0.26	0.89	
Spectrum Emmental	7.41	0.04	1.41	0.15	

Notes:
msc : Maximum Squared Coherency

continued on next page

Table 3 continued

	Period	*msc*	Phase	Gain	VAR-Order
Waadt/Aargau					
Max. of Coh.	10.64	0.04	-4.75	0.26	
	3.65	0.38	0.06	0.59	
Spectrum Waadt	11.24	0.04	-4.03	0.27	
	6.45	0.01	0.95	0.09	
	2.87	0.25	-0.29	0.48	
Spectrum Aargau	5.08	0.21	0.35	0.36	
	2.81	0.24	-0.28	0.47	
Waadt/Zurich					
Max. of Coh.	6.54	0.27	1.09	0.45	
	2.91	0.43	0.15	0.77	
Spectrum Waadt	11.24	0.09	0.31	0.35	
	6.45	0.27	1.08	0.45	
Spectrum Zurich	6.25	0.26	1.06	0.44	
	2.79	0.40	0.16	0.71	
Seeland/Region Bern					
Max. of Coh.	7.04	0.57	-0.49	0.86	
	3.53	0.26	0.31	0.83	
Spectrum Seeland	6.90	0.57	-0.44	0.85	
	4.72	0.20	0.31	0.61	
	2.81	0.38	0.05	1.30	
Spectrum Bern	5.92	0.44	-0.10	0.64	
	2.61	0.48	-0.04	1.12	
Seeland/Emmental					
Max. of Coh.	7.75	0.43	0.56	0.51	
	2.60	0.40	-0.18	1.03	
Spectrum Seeland	6.90	0.36	0.59	0.51	
	4.72	0.00	-0.50	0.09	
Spectrum Emmental	7.41	0.42	0.59	0.52	

Notes:
msc : Maximum Squared Coherency

continued on next page

Table 3 continued

	Period	*msc*	Phase	Gain	VAR-Order
Seeland/Aargau					
Max. of Coh.	4.03	0.32	0.13	0.47	
	2.81	0.29	-0.40	0.57	
Spectrum Seeland	6.90	0.04	3.69	0.19	
	4.72	0.28	-0.05	0.44	
	2.81	0.29	-0.40	0.57	
Spectrum Aargau	5.08	0.22	-0.17	0.37	
	2.81	0.29	-0.40	0.57	
Seeland/Zurich					
Max. of Coh.	11.36	0.31	1.12	0.42	
	6.85	0.28	0.29	0.50	
Spectrum Seeland	6.90	0.28	0.30	0.51	
	4.72	0.05	0.13	0.26	
	2.81	0.58	0.13	0.96	
Spectrum Zurich	6.25	0.25	0.25	0.44	
	2.79	0.57	0.12	0.95	
Region Bern/Emmental					
Max. of Coh.	7.63	0.47	1.13	0.44	
	5.15	0.20	-0.55	0.44	
Spectrum Bern	5.92	0.20	0.18	0.45	
	2.61	0.35	-0.15	0.60	
Spectrum Emmental	7.41	0.47	1.08	0.46	
Region Bern/Aargau					
Max. of Coh.	3.30	0.24	0.11	0.25	
	2.49	0.39	-0.21	0.40	
Spectrum Bern	5.92	0.03	-0.69	0.15	
	2.61	0.37	-0.26	0.37	
Spectrum Aargau	5.08	0.07	-0.44	0.19	
	2.81	0.26	-0.19	0.26	

Notes:
msc : Maximum Squared Coherency

continued on next page

Table 3 continued

	Period	*msc*	Phase	Gain	VAR-Order
Region Bern/Zurich					
Max. of Coh.	6.29	0.19	0.22	0.38	
	2.20	0.35	-0.03	0.31	
Spectrum Bern	5.92	0.17	0.12	0.37	
	2.61	0.31	-0.03	0.38	
Spectrum Zurich	6.25	0.19	0.21	0.38	
	2.79	0.22	-0.02	0.29	
Emmental/Aargau					
Max. of Coh.	7.25	0.10	-4.86	0.37	
	3.53	0.13	-0.41	0.19	
Spectrum Emmental	7.41	0.10	-4.96	0.37	
Spectrum Aargau	5.08	0.04	0.33	0.15	
	2.81	0.10	-0.20	0.15	
Emmental/Zurich					
Max. of Coh.	10.00	0.05	1.55	0.26	
	6.54	0.10	-0.76	0.32	
Spectrum Emmental	7.41	0.06	-0.48	0.29	
Spectrum Zurich	6.25	0.10	-0.70	0.30	
	2.79	0.41	-0.02	0.37	
Aargau/Zurich					
Max. of Coh.	3.86	0.32	0.10	0.80	
	2.92	0.20	0.56	0.54	
Spectrum Aargau	5.08	0.18	-0.07	0.56	
	2.81	0.19	0.57	0.52	
Spectrum Zurich	6.25	0.06	0.17	0.25	
	2.79	0.18	0.55	0.51	

Notes:
msc :Maximum Squared Coherency

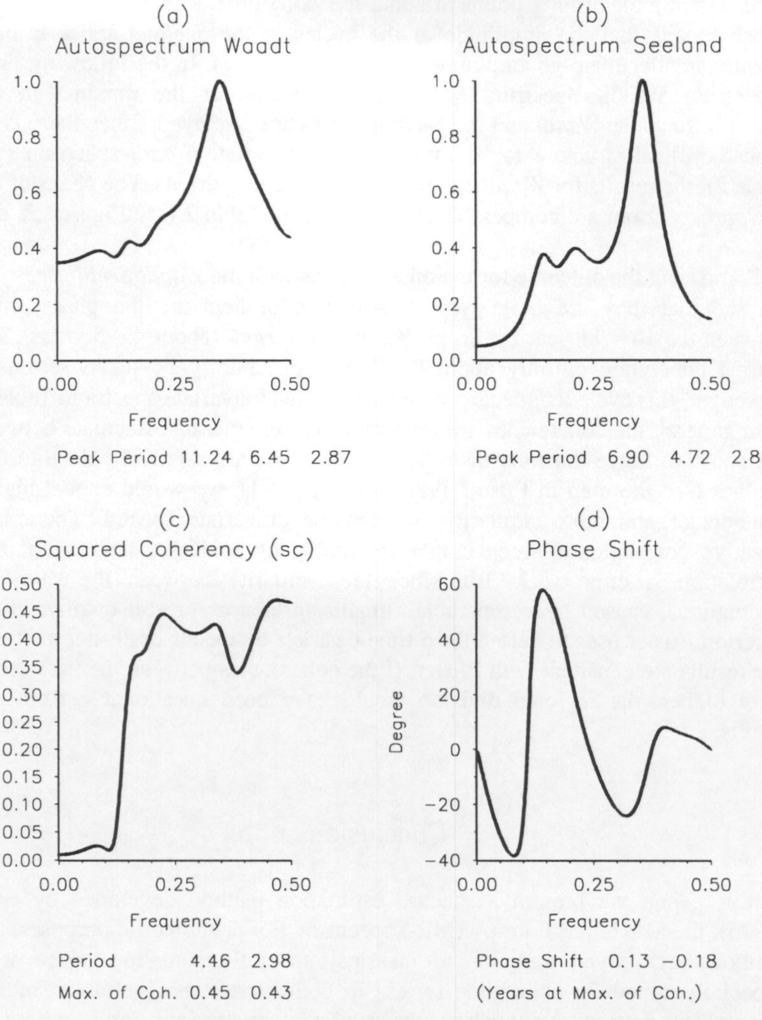

Figure 5: Waadt/Seeland: Multivariate Spectra, HP-filtered Series

annual data, we can conclude that the cycles in the *Waadt* and the *Seeland* series reach their turning points at about the same time. The gain is about 0.7, which means that the amplitude of the cycles in the *Seeland* series is only slightly smaller than the amplitudes in the *Waadt* series. In the following lines (Spectrum Waadt, Spectrum Seeland) the results for the maxima in the autospectra of the *Waadt* and the *Seeland* series are displayed. Since there is no robust cyclical structure to be found in the univariate *Waadt* spectrum (see Table 2), the results for *Waadt* can be looked upon as spurious. The peaks of the *Seeland* spectrum are comparable to the results in Table 2 (see Figures 5a and 5b).

Comparing the outcome for the other regions with the *WaadtlSeeland* results, we see that they are quite typical. Apparent outliers are the phase shifts between the 10 - 11 year cycles in *Waadt* and *Aargau* (about 4 - 5 years). But with a coherency of only about 0.04, they can hardly be taken seriously. Moreover, this cycle length cannot be found in the univariate spectra in Table 2.

In general, the coherencies are relatively low; the maxima fluctuate between 0.2 and 0.6. Since the tithe districts analyzed here are relatively close to one another (see the map in Pfister 1985, Vol. II, p. 71), we would expect higher coherencies and more similarity between the univariate spectra. There is a negative correlation beween coherency and geographical distance, but this correlation is only weak. But since the similarity between the short-run fluctuations, caused by comparable climatic influences or soil quality, is the criterion Pfister uses to define the 6 tithe destricts by means of cluster analysis, our results are consitent with Pfister. If the coherencies between the tithe series were higher, the regional division would have been questionable from our results.

Conclusion

In this paper, we present a spectral estimation method developed by Burg (1975), the Maximum Entropy (ME-) spectrum. For econometric purposes, this method has proved superior to classical spectral estimation, since it is especially suited for short time series. To demonstrate the usefulness of ME spectral estimation, we analyze the cyclical structure of grain output in Mid-Franconia and Switzerland.

The main result of the study is that the cyclical structure of grain output measured by the tithe is dominated by a short cycle with a length of 2-4 years. Since these cycles can be found in both the HP- and the difference filtered series, we can be relatively sure that the cycles are not spurious. The similarity between the cycles in Mid Franconia and Switzerland is remarkable, given the distance between these regions and the fact that the observation period is different (1339-1630 for Mid Franconia, 1558-1708 for Switzerland).

There are obvious extensions of the above analysis: First, the interrelation between output fluctuations and grain price cycles can be analyzed Bauernfeind and Woitek (1994). Second, the influence of climatic changes on the tithe fluctuations could be analyzed to the extent that data are available. The existence of weather cycles being generally accepted, it would be interesting to analyze the dependence of the harvest cycle found in the Mid Franconian and the Swiss tithe series on climatic cycles. But this topic must be left for further research.

References

Abies, J. G., "Maximum Entropy Spectral Analysis," *Astronomy and Astrophysics Supplement Series, 1974, 15,* 383-393. Reprinted in Childers (1978), pp. 23-33.

Backus, D.K. and P. J. Kehoe, "International Evidence on the Historical Properties of Business Cycles," *American Economic Review, 1992, 82,* 864-888.

Bauernfeind, W., *Materielle Grundstrukturen im Spätmittelalter und der Frühen Neuzeit - Preisentwicklung und Agrarkonjunktur am Nürnberger Getreidemarkt von 1339-1670,* Nürnberg: Universitätsbuchhandlung Korn und Berg, 1993.

Bauernfeind, W. and U. Woitek, "Business Cycles in Germany 1339-1670: A Spectral Anaylsis of Grain Prices and Production in Nuremberg," 1994. Münchener Wirtschaftswissenschaftliche Beiträge Nr. 94-19.

Bengtsson, T. and L. Jörberg, "Market Integration in Sweden during the 18th and 19th Centuries. Spectral Analysis of Grain Prices," *Economy and History, 1975, XVIII,* 93-106.

Bengtsson, T. and R. Ohlsson, "Population and Economic Fluctuations in Sweden 1749-1914," in T. Bengtsson, G. Fridlizius, and R. Ohlsson, eds., *Pre-Industrial Population Change. The Mortality Decline and Short-Term Population Movements,* Stockholm: Almquist and Wiksell International, 1984, pp. 277-297.

Beveridge, W. H., "Weather and Harvest Cycles," *Economic Journal, 1921,31,* 429-452.

Brandner, P. and K. Neusser, "Business Cycles in Open Economies: Stylized Facts for Austria and Germany," *Weltwirtschaftliches Archiv, 1992, 128,* 67-87.

Brockwell, P. J. and R. A. Davis, *Time Series: Theory and Methods,* second ed., Berlin, Heidelberg, New York, Tokio: Springer, 1991.

Burg, J. P., "Maximum Entropy Spectral Analysis." PhD dissertation, Stanford University 1975.

Cargill, T. F., "Early Applications of Spectral Methods to Economic Time Series," *History of Political Economy, 1974, 6,* 1-16.

Chan, K. H., J. C. Hayya, and J. K. Ord, "A Note on Trend Removal Methods: The Case of Polynomial Regression Versus Variate Differencing," *Econometrica, 1977, 45,* 737-744.

Childers, D. G., ed., *Modern Spectrum Analysis,* IEEE Press: New York, 1978.

Crum, W. L., "Cycles of Rates on Commercial Paper," *The Review of Economics and Statistics, 1923, 5,* 17-27.

Davis, H. T., *The Analysis of Economic Time Series,* San Antonio, Texas: Principia Press, 1941. (Reprinted 1963).

Dickey, D. A., "Estimation and Hypothesis Testing in Nonstationary Time Series." PhD dissertation, Iowa State University 1976.

Dickey, D. A. and W. A. Fuller, "Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root," *Econometrica, 1981, 49,* 1057-1072.

Dolado, J. J., T. Jenkinson, and S. Sosvilla-Rivero, "Cointegration and Unit Roots," *Journal of Economic Surveys, 1990, 4,* 249-273.

Eisner, M., "Long-Term Fluctuations of Economic Growth and Social Destabilization," *Historical Social Research, 1992,17,* 70-98.

Fougere, P. F., "A Review of the Problem of Spontaneous Line Splitting in Maximum Entropy Power Spectral Analysis," in C. R. Smith and W. Grandy, eds., *Maximum-Entropy and Bayesian Methods in Inverse Problems,* Dordrecht: D. Reidel Publishing Company, 1985, pp. 303-314.

Fuller, W. A., *Introduction to Statistical Time Series,* New York, Chichester, Brisbane, Toronto, Singapore: John Wiley & Sons, 1976.

Goy, J. and E. Le Roy Ladurie, *Les Fluctuations du Produit^ de la Dime. Conjuncture Décimale et Domaniale de la Fin du Moyen Age au XVIII' Steele,* Paris: Mouton, 1972.

Goy, J. and E. Le Roy Ladurie, *Tithe and Agrarian History from the Fourteenth to the Nineteenth Centuries. An Essay in Comparative History,* Cambridge: Cambridge University Press, 1982.

Greenstein, B., "Periodogram Analysis with Special Application to Business Failures in the United States, 1867-1932," *Econometrica, 1935, 3,* 170-198.

Harvey, A. C, *The Econometric Analysis of Time Series,* second ed., New York, London, Toronto, Sydney, Tokyo, Singapore: Philip Allan, 1990.

Harvey, A. C. and A. Jaeger, "Detrending, Stylized Facts and the Business Cycle," 1991. London School of Economics, Discussion Paper No.EM/91/230.

Heintel, M., "A Bayesian Way to Identify the Order of Autoregressive Processes," 1994. Münchner Wirtschaftswissenschaftliche Beiträge.

Hillinger, C. and M. Sebold-Bender, "The Stylized Facts of Macroeconomic Fluctuations," in C. Hillinger, ed., *Cyclical Growth in Market and Planned Economies,* London: Oxford University Press, 1992.

Hodrick, R. and E. Prescott, "Postwar U.S. Business Cycles: An Empirical Investigation," 1980. Discussion Paper No. 451, Carnegie-Mellon University.

Judge, G. G., R. C. Hill, W. E. Griffiths, H. Lütkepohl, and L. Tsuong-Chao, *Introduction to the Theory and Practice of Econometrics,* second ed., New York, Chichester, Brisbane, Toronto, Singapore: John Wiley & Sons, 1988.

King, R. G. and S. T. Rebelo, "Low Frequency Filtering and Real Business Cycles," *Journal of Economic Dynamics and Control, 1993, 17,* 207-231.

Koopmans, L. H., *The Spectral Analysis of Time Series,* New York, San Francisco, London: Academic Press, 1974.

Ljung, G. M. and G. E. P. Box, "On a Measure of Lack of Fit in Time Series Models," *Biometrika, 1978, 66,* 67-72.

Lütkepohl, H., "Comparison of Criteria for Estimating the Order of a Vector Autoregressive Process," *Journal of Time Series Analysis, 1985, 6.*

Lütkepohl, H., *Introduction to Multiple Time Series Analysis,* Berlin, Heidelberg, New York, Tokio: Springer, 1991.

Marple, S. L., *Digital Spectral Analysis with Applications,* Englewood Cliffs: Prentice Hall, 1987.

Marple, S. L. and A. H. Nuttal, "Experimental Comparison of Three Multichannel Linear Prediction Spectral Estimators," *IEEE PROC, 1983, 130,* 218-229.

Metz, R., "Ansätze, Begriffe und Verfahren der Analyse ökonomischer Zeitreihen," *Historical Social Research (Historische Sozialforschung), 1988, 13,* 23-103.

Moore, H. L., *Economic Cycles - Their Law and Their Cause,* New York: Macmillan, 1914.

Moore, H. L., *Generating Economic Cycles,* New York: Macmillan, 1923.

Morf, M., A. Vieira, D. T. Lee, and T. Kailath, "Recursive Multichannel Maximum Entropy Spectral Estimation," *IEEE Transactions on Geoscience Electronics.*

Nelson, C. R. and H. Kang, "Spurious Periodicity in Inappropriately Detrended Time Series," *Econometrica, 1981, 49,* 741-751.

Neveux, H., "Die langfristigen Bewegungen der französischen Getreideproduktion vom 14.-18. Jahrhundert," *Scripta Mercaturae, 1979, 13,* 75-88.

Perron, P., "The Great Crash, the Oil Price Shock, and the Unit Root Hypothesis," *Econometrica, 1989, 57,* 1361-1402.

Pfister, C, *Klimageschichte der Schweiz 1525-1860. Das Klima der Schweiz und seine Bedeutung in der Geschichte der Landwirtschaft, 2* ed., Vol. I and II, Bern: Paul Haupt, 1985.

Priestley, M., *Spectral Analysis and Time Series,* London: Academic Press, 1981.

Rösener, W., "Krisen und Konjunkturen der Wirtschaft im spätmittelalterlichen Deutschland," in F. Seibt and W. Eberhard, eds., *Europa 1400. Die Krise des Spätmittelalters,* Stuttgart Klett-Cotta, 1984, pp. 24-38.

Rudebusch, G. D., "Trends and Random Walks in Macroeconomic Time Series: A Re-Examination," *International Economic Review, 1992, 33,* 616-680.

Rudebusch, G. D., "The Uncertain Unit Root in Real GDP," *American Economic Review, 1993, 83,* 264-272.

Schuster, A., "On the Investigation of Hidden Periodicities with Application to a Supposed Twenty-Six-Day Period of Meteorological Phenomena," *Terrestrial Magnetism, 1898, 3,* 13-41.

Swingler, D., "A Comparison Between Burg's Maximum Entropy Method and a Nonrecursive Technique for the Spectral Analysis of Deterministic Signals," *Journal of Geophysical Research, 1979, 84,* 679-685.

Wilson, E. B., "The Periodogram of American Business Activity," *Quarterly Journal of Economics, 1934, 48,* 375-417.

Woitek, U., "Business Cycles. An International Comparison of Stylized Facts in a Historical Perspective." PhD dissertation, Economics Department, University of Munich 1995.

Woitek, U., "The G7-Countries: A Multivariate Description of Business Cycle Stylized Facts," in W. Barnett, G. Gandolfo, and C. Hillinger, eds., *Dynamic Disequilibrium Modelling: Theory and Applications,* Cambridge: Cambridge University Press. To be published.